An origin of the Universe determined by quantum physics and relativistic gravity

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September 28, 2001

Abstract

We discuss the evolution of the Universe from what might be called its quantum origin. We apply the uncertainty principle to the origin of the Universe with characteristic time scale equal to the Planck time to obtain its initial temperature and density. We establish that the subsequent evolution obeying the Einstein equation gives the present temperature of the microwave background close to the observed value. The same origin allows the possibility that the Universe started with exactly the critical density, $\Omega = 1$, and remained at the critical density during evolution. Many other important features of the observed Universe, including homogeneity and isotropy, Hubble's constant at origin, its minimum age, present density etc. are all predictions of our theory. We discuss also the testability of our theory.

The present standard model of cosmology relies on a big bang origin of the Universe in which reverse evolution via the Einstein equations yields a formal singularity. There are physical reasons to believe that the birth of the Universe was controlled by quantum processes which might avoid such a singularity, but a satisfactory theory of quantum gravity is still to be found.

In this paper we avoid the necessity of a formal quantum gravity theory to describe the origin of the Universe by making the assumption that the quantum origin would obey the uncertainty principle and relativistic gravity in an essential way. This allows us to deduce some important conclusions regarding observable features of the present Universe.

Quantum processes such as state life times, tunneling etc. are characterized by the energy-time uncertainty relation, $\Delta E \Delta t \geq \hbar/2$. In the present discussion we are concerned with a physical system in its lowest energy state and the energy itself serves as ΔE.

Our fundamental hypothesis is that the Universe took a quantum birth with a characteristic time scale of the Planck time, the time scale being fixed by quantum physics and relativistic gravity. (This time scale arises naturally if the origin involves crossing above a gravitational barrier, classically forbidden. Then the barrier height is given by a condition like $2G(E/c^2)/c^2 r \simeq 1$, with $r = ct$ and a second equation is provided by the quantization condition $Et \simeq \hbar/2$. These can be solved for $t = (\hbar G/c^5)^{1/2} \equiv t_P \simeq 5.4 \times 10^{-44}$ s). Then we get the initial average energy as

$$E_i \simeq \hbar/2t_P \tag{1}$$

At the end of the creation process the Universe is filled with radiation and relativistic constituents with equivalent temperature given by the average energy divided by the Boltzmann constant,

$$T_i = E_i/k_B \simeq 7 \times 10^{31}\ {}^\circ K \tag{2}$$

In effect, our assumptions have fixed the initial temperature of the Universe to be approximately the Planck temperature. This value is a minimum temperature consistent with the uncertainty principle.

We expect that in the beginning an energy content of E_i will be present in a volume scaled to the Planck length, if our theory is consistent. The scale comes into picture through the causal horizon at Planck time, $r_H \simeq ct_P$. We do not make any assumption on the actual size of the Universe at origin, since this is not a well defined concept in general, though the observable Universe is finite at any epoch. Starting from the quantum mechanical estimate, we get for the initial energy density

$$\rho_i = \frac{1}{2}(\frac{\hbar c^5}{G})^{1/2}/\frac{4\pi}{3}(\frac{\hbar G}{c^3})^{3/2} = \frac{3}{8\pi}\frac{c^7}{\hbar G^2} \tag{3}$$

This is in fact the critical density as we will show later. We assume that for larger time scales General Relativity is valid, at least to a good approximation, and evolve the temperature and density forward in time using the Einstein's equations.

The equations governing the evolution of the scale factor and the density in the Universe are

$$\dot{R}^2 + k = \frac{8\pi G}{3}\rho R^2 \tag{4}$$

$$\frac{d}{dR}(\rho R^3) = -3pR^2 \tag{5}$$

The equation of state $p = p(\rho)$ will allow the determination of the evolution of the density as a function of the scale factor. With this, $R(t)$ can be determined from eq. 3. Here we consider a Universe that has been radiation dominated from the beginning to until about the time $t \simeq 10^{12}$ s, and then matter dominated until present, $t_0 \simeq 5 \times 10^{17}$ s [1, 2]. Since the initial density is estimated to be the critical density we set $k = 0$.

During the radiation dominated era the scale factor varies as

$$R(t) \sim t^{1/2} \tag{6}$$

The temperature of the radiation and relativistic particles varies as the inverse of the scale factor.

$$T(t) \sim 1/R(t) \sim t^{-1/2} \tag{7}$$

Therefore, at the end of the radiation dominated era ($t_r \simeq 10^{12}$ s), we have for the temperature,

$$T(t_r) = T_i/(t_r/t_P)^{1/2} \simeq 5 \times 10^4 \ {}^\circ K \tag{8}$$

Near the end of this era the energy density of the relativistic particles and radiation falls below that of the density of Hydrogen and Helium in the

Universe. The subsequent evolution of the temperature is slightly faster, since the scale factor varies as $t^{2/3}$ during the matter dominated era, whereas the temperature still varies as the inverse of the scale factor. So, during $t \simeq 10^{12}$ s to the present, $t_0 \simeq 5 \times 10^{17}$ s,

$$T(t) \sim 1/R(t) \sim t^{-2/3} \tag{9}$$

$$T(t_0) = T(t_r)/(t_0/t_r)^{2/3} \simeq 1.7\ {}^\circ K \tag{10}$$

This prediction of the present temperature, starting from an initial temperature determined by the quantum mechanical origin we have chosen and the constants relevant to quantum physics and relativistic gravity, is remarkably close to the observed present temperature. The temperature of the microwave background will be a factor of about 1.4 higher from our estimate due to the entropy production in $e^+ - e^-$ annihilation[1, 2], which yields a final value of 2.4° K. A number of other factors could slightly alter our estimate, such as the lack of precision in knowledge of the age of the Universe, and the accurate use of the transition from the radiation to the matter dominated era. Even the presence of a cosmological constant that dominates the energy density during the relatively recent evolution of the Universe would not change our estimate considerably.

While the temperature of the background radiation is an important touchstone for any hypothesis about the birth and the evolution of the Universe, it is desirable that the hypothesis is tested with other comparisons that are significant. It turns out that the Universe can start off naturally with the critical density if its origin is in the quantum process we are discussing. This feature is very attractive since inflation is the only other known mechanism that can endow the Universe with critical density naturally, without drastic fine tuning.

We had estimated above that the energy density at origin is $\rho_i = \frac{3}{8\pi}\frac{c^7}{\hbar G^2}$. This is the critical energy density as seen directly from the Friedman equation,

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = \frac{8\pi G\rho}{3} \tag{11}$$

For a Universe at critical density (flat), $k = 0$. At the origin we have

$\dot{R}/R = c/ct_P = 1/t_P$. Then we have

$$\frac{1}{t_P^2} = \frac{8\pi G\rho_c}{3} \tag{12}$$

$$\rho_c = \frac{3}{8\pi}\frac{c^5}{\hbar G^2} \tag{13}$$

The critical energy density is $\rho_c c^2$, which is just what we have calculated from the quantum origin scenario. The density evolves as $1/t^2$ during the radiation dominated and the matter dominated era and this gives the present (critical) energy density.

$$\rho_0 = \rho_i/(\frac{t_0}{t_P})^2 = \frac{3c^2}{8\pi G t_0^2} \tag{14}$$

It is important that this can be obtained only if we use the time scale t_P in the uncertainty principle. If we use the uncertainty principle at any other epoch, arbitrarily chosen, we do not get the density of the Universe as the critical density, nor the correct present temperature. This should be considered far from coincidence.

This theory solves a fundamental enigma about the observed Universe. It is well known that if the present Universe has its density of the order of the critical density then in the past the density should have been even closer to the critical density. The ratio $\Omega = \rho/\rho_c$ moves closer and closer to unity as $|\Omega - 1| \sim (1+z)^{-2}$ in the past[2]. At Planck epoch this ratio should have been unity accurate to a part in $\approx 10^{60}$. This kind of fine tuning was beyond explanation till present, except in an inflationary scenario for which the present density is the critical density irrespective of the pre-inflationary era.

We note that the quantity $2GM/c^2 r_H$ at the quantum birth is unity, where M is the mass contained in a volume of radius equal to the horizon size. $E_i t_P \simeq \hbar/2$ for the quantum birth condition gives $E_i \simeq M_P c^2/2$. We get

$$2GM/c^2 r_H = GM_P/c^2 L_P = 1 \tag{15}$$

It is interesting to note that the present observable Universe also approximately obeys this relation. The horizon size changes linearly in t, and the present horizon size is $r_0 \simeq ct_0 \simeq 10^{28} cm$. The scale factor itself increases at a slower rate, and the primordial elementary volume of Planck length scale would have evolved into a size of only about 10^{-2} cm at present.

We now show that the quantity $GM_H/c^2 r_H$ is preserved at its initial value. The density of the Universe varies as $\rho(t) \sim 1/t^2$ at all times and the horizon size increases as $r_H \sim t$. The mass contained in horizon size r_H is

$$M_H \sim \rho(t) r_H^3 \sim t \tag{16}$$

This gives

$$GM_H/c^2 r_H = \text{constant} \tag{17}$$

Another significant feature is that the quantum origin and the present temperature of the microwave background fix a lower limit to the age of the Universe. Since the initial temperature we estimated was the minimum temperature, the time duration for the unique trajectory connecting the initial temperature and the present temperature gives the minimum age of the Universe, and it is about 15 billion years. Our hypothesis also fixes Hubble's constant at the origin independently as $H_i = \dot{R}\ /R = c/ct_P = 1/t_P \simeq 1.9 \times 10^{43}$ s^{-1}. Since $\dot{R}\ /R \sim 1/t$ subsequently, for any power law evolution of R, the present value of Hubble parameter would be

$$H_0 = \frac{1}{t_P}(\frac{t_P}{t_0}) = 1/t_0 \tag{18}$$

Combined with the age determined as earlier this gives the present Hubble parameter as about 67 km/s/Mpc. (Several recent measurements of the Hubble constant have been reported. Gibson et al., [3] for instance, find $H_0 = 68 \pm 2 \pm 5$ km s^{-1}Mpc^{-1}, where the first uncertainty value is the random component and the second one is the systematic component. The general range into which the recent measurements are converging [4] is 60 to 75 km s^{-1}Mpc^{-1}.) The deceleration parameter at origin is also determined since $q_0 = \Omega$ during the initial epoch when the Universe was radiation dominated.

Now we bring out another important conclusion from our theory. Since the whole of the presently observable Universe is causally linked to the primordial Planck volumes in which the same fundamental constants determined the initial conditions, this theory predicts homogeneity and isotropy of the bare Universe, and all deviations from such a state have to occur through structure formation. The presently observable Universe of size scale 10^{28} cm would have been of size scale 10^{-3} cm at Planck epoch (due to dominantly $t^{1/2}$ evolution of size scale) [2]. We see that though the horizon size at the Planck time is much smaller than the total size of the Universe, by a factor of about 10^{30}, homogeneity and isotropy over the whole Universe is assured at early times. Thus homogeneity and isotropy is the signature of homogeneity and isotropy of the fundamental constants.

If the hypothesis is fundamentally correct then the spatial variation and anisotropy of G, c and $\hbar$ are constrained very precisely. Any spatial variation of these constants from the values that determined the critical density at Planck time would lead to regions of space that are very far away from critical density at present. Since the total density contrast at all scales at the end of the radiation dominated epoch is limited to about 10^{-5}, and since $|\Omega - 1| \sim (1+z)^{-2}$ during earlier times [2], the allowed spatial variation can be estimated to be about 10^{-60} at origin over a length scale larger than Planck length. The length scale of each primordial elementary volume at present is about 10^{-2} cm. The significance of this is that laboratory observations or astrophysical observations that can confirm a spatial variation or anisotropy of even such a tiny amount over their accessible scales will test our theory by a large margin.

Since the present temperature of the microwave background is precisely determined, the theory requires some level of temporal constancy of the fundamental constants G, c, and h. It turns out that a fractional variation of about 10^{-11}/year or smaller is consistent with our theory.

The validity of our hypothesis can have very interesting and useful implications for the theoretical understanding of the evolution of the Universe. In effect, it fixes two points in the history of the evolution of the Universe, its birth and the present, and the possible evolution trajectories consistent with various observations can be very limited. This should be contrasted with Big Bang cosmology in which various physical quantities at the origin are infinite or zero. The two precisely known points in the thermal history we have outlined encompass all the interesting aspects of particle physics during the evolution of the Universe, from String physics to low energy physics.

We conclude that our theory and analysis provides significant evidence that the initial conditions for the evolution of the Universe from a nonsingular origin was fixed by the fundamental constants of quantum physics and relativistic gravity and that the observed background radiation and the present density close to the critical density are the relics of the quantum birth of the Universe.

Acknowledgments: We thank Ramanath Cowsik for discussions on entropy production in the Universe and on other aspects of particle thermodynamics. Laboratoire Kastler Brossel is Unité de Recherche de l'Ecole Normale Supérieure et de l'Université Pierre et Marie Curie, associée au CNRS. The non-accelerator particle physics program at the Indian Institute of Astrophysics is supported by the Department of Science & Technology, Government of India.

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